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Grading Entropy based Criterion for Assessing the Potential of Internal Instability of Granular Soils.

Gang Zhang, Buddhima Indraratna & Jahanzaib Israr
Centre for Geomechanics and Railway Engineering (Centre GRE),
ARC Industry Transformation Training Centre for Advanced Rail Track
Technologies (ARC ITTC) School of Civil, Mining and Environmental Engineering, University of
Wollongong, Wollongong City, NSW 2522, Australia. E-mail (Corresponding Author): jisrar@uow.edu.au

ABSTRACT

Internal instability is a phenomenon whereby seepage induced internal erosion of finer fraction occurs in granular soils to an extent to cause permanent changes in their original particle size distributions. Hitherto, various empirical criterion for assessing the potential of instability have been proposed that demarcate the original particle size distribution curve of subject soil to obtain arbitrary coarser (i.e. filter) and finer (base) fractions and apply some well-accepted filter criterion to examine if the former can retain the latter. However, there is no universal agreement on how to demarcate the above division point as well as none of the existing methods provide 100% correct information on instability potential of soils. In this study, a grading entropy based internal instability criterion is proposed for assessing the instability potential of soils. In essence, the information from particle size distribution curve can be described in the normalized entropy diagram using the concept of grading entropy. The entropy of select soil gradations characterized as stable and unstable could be plotted in the normalized entropy diagram and differences between these curves and subject optimal soil gradations were analyzed using the principle of maximum entropy. A large body of published experimental data could be successfully examined for its correct potential of instability compared to two well-accepted existing methods that sufficiently verified the rigor of this new entropy based criterion. Moreover, the proposed criterion formulated on the basis of firm scientific understanding of the phenomenon could successfully capture the true potential of instability of most narrowly-graded, well-graded, gap-graded and widely-graded soils using only two normalized entropy coordinates of a given particle size distribution curve.

RÉSUMÉ

L'instabilité interne est un phénomène par lequel l'érosion interne induite par infiltration de la fraction plus fine se produit dans les sols granulaires dans une mesure qui provoque des changements permanents dans leurs distributions granulométriques d'origine. Jusqu'à présent, différents critères empiriques pour évaluer le potentiel d'instabilité ont été proposés pour délimiter la courbe de distribution granulométrique originale du sol sujet afin d'obtenir des fractions arbitraires plus grossières (filtre) et plus fines (base) et appliquer un critère de filtre bien accepté. le premier peut retenir le dernier. Cependant, il n'y a pas d'accord universel sur la façon de délimiter le point de division ci-dessus, et aucune des méthodes existantes ne fournit une information correcte à 100% sur le potentiel d'instabilité des sols. Dans cette étude, un critère d'instabilité interne basé sur l'entropie de granulométrie est proposé pour évaluer le potentiel d'instabilité des sols. En substance, l'information provenant de la courbe de distribution granulométrique peut être décrite dans le diagramme d'entropie normalisé en utilisant le concept d'entropie de granulométrie. L'entropie de certaines granulométrie de sol caractérisées comme stables et instables pourrait être tracée dans le diagramme d'entropie normalisé et les différences entre ces courbes et les granulométrie optimales du sol ont été analysées en utilisant le principe de l'entropie maximale. Un grand nombre de données expérimentales publiées pourraient être examinées avec succès pour déterminer leur potentiel d'instabilité par rapport à deux méthodes existantes bien acceptées qui ont suffisamment vérifié la rigueur de ce nouveau critère basé sur l'entropie. De plus, le critère proposé formulé sur la base d'une compréhension scientifique ferme du phénomène pourrait bien saisir le véritable potentiel d'instabilité des sols les plus gradués, bien classés, classés et bien classés, en utilisant seulement deux coordonnées d'entropie normalisées d'un donnée courbe de distribution de la taille des particules.

1. INTRODUCTION

Seepage induced internal erosion in granular soils were observed to be the major causes of failures of hydraulic structures worldwide, i.e. up to 50% of total failures so far (Israr et al, 2016; Richards and Reddy 2007). These failures were recognized as internal instability phenomena in to the form of contact erosion, backward erosion and/or concentrated leak, whereby the finer particles erode through the soil skeleton causing mutation of hydraulic characteristics and the reduction of shear strength under seepage flow (Indraratna et al. 2017; Israr, 2016; Chang and Zhang, 2013). A soil which is susceptible to loss of its finer fraction is termed as internally unstable. Instability in soils would be governed by specific combinations of their geometric and hydromechanical characteristics such as particle size distribution (PSD), constriction size distribution (CSD), external loading, critical hydraulic gradients and associated levels of effective stress. Geometric and mechanical conditions affect the potential of internal instability whereas hydraulic conditions govern the onset of any instability (Chang and Zhang, 2013; Indraratna, et al. 2015). Nevertheless, most dam failures occurred due to the absence of an adequate filter to protect the internally unstable base soils. Thus far, various geometrical assessment methods have been proposed to investigate the instability of soils based on PSD and CSD of soils, where the latter would also incorporate the effects of compaction (e.g. Sherard 1979; Kenney and Lau 1985; Burenkova 1993; Indraratna et al. 2011; Indraratna et al. 2015; Israr and Indraratna 2017). This paper purports to examine the potential of internal instability of granular soils based on their PSD curves.

As a pioneer, USACE (1953) investigated the "inherent stability" of a soil mixture of sand and gravel to find its own ability to resist the occurrence of segregation and piping. Istomina (1957) suggested an internal stability criterion for granular soils using the coefficient of uniformity, C_u, and proposed an internal stability criterion based on Terzaghi's filter rule. For completeness, this criterion divides the particle size distribution (PSD) curve into finer (erodible particles) and coarser (stable particles) fractions. Notably, Sherard (1979) and Kezdi (1979) independently proposed criterion similar to that of Istomina (1957).

Later on, Kenney and Lau (1985) introduced an internal stability index $(H/F)_{min}$ obtained from particle size distribution curve to examine the internal instability potential of soil (F = the mass fraction finer than particle size *d* and H = the mass fraction between particle size d and 4*d*). Indraratna et al. (2011) extended the constriction based retention criterion of Raut and Indraratna (2008) to evaluate the potential of suffusion in granular soils. Indraratna et al. (2015) combined the criterion of Kenney and Lau with the controlling constriction model of Indraratna et al. (2007) to capture the effects of soil's relative density (R_d) and proposed a more accurate constriction size distribution (CSD) based method to assess the potential of internal instability. Nevertheless, the determination of CSD of a soil would require complicated programs or experimental procedures to follow that may be the reason why PSD based methods are still very popular in practice (Indraratna et al. 2016; Israr and Indraratna 2017).

The concept of grading entropy was first proposed by Lőrincz (1986), whereby the information of particle grading curve can be expressed properly as a group of parameters using grading entropy theory, which is already applied in the research areas of dry bulk density and separation processes of granular materials, the change of grading curves due to soil crushing, and stability criterion for piping and segregation (Lőrincz 1990; Lőrincz et al. 2015). This current study deals with the assessment of internal instability potential based on soil's PSD curve. Adopting the concept of optimal soil gradation based on the principle of maximum entropy, a novel grading entropy based simple but effective procedure has been proposed for prompt assessments of internal instability potential. In addition, the proposed method was examined for a large published dataset that showed good agreement with the experimental results.

2. GRADING ENTROPY METHOD

The particle size distribution (PSD) curve is a fundamental parameter for granular materials that contains significant information such as particle sizes, constriction sizes, as well as entropy distribution etc. The entropy is a quantity of the theory of probability and is determined by the following equation:

$$S = -\sum_x p(x) \log_2 p(x) \tag{1}$$

According to the equation (1), the particle size distribution curve can be divided into several statistical cells, and then the specific entropy can be given in the following form:

$$s = -\sum_x \alpha_i \log_2 \alpha_i \tag{2}$$

where α_i is the frequency of the *i*-th statistical cell. In order to use the statistical entropy theory to express the distribution of particles, a double statistical cell system is used (Lőrincz 1986). The PSD curve is discretized into several fractions with the sizes as a 2 multiplier geometric series (e.g. *d*=0.0625, 0.125, 0.5, 1, 2 mm), which is the same as a mechanical sieve analysis (Figure 1). Each fraction is also discretized by the minimum grain diameter into an imaginary elementary cell system and the limiting *d* values for the *i*th fraction in terms of d_{min} can be expressed as follows:

$$2^i d_{\min} \le d_i \le 2^{i+1} d_{\min} \tag{3}$$

Each fraction has a relative frequency x_i, which corresponding to value of weight percent in soil grading curve, it concludes that (Lőrincz et al 2015)

$$\sum_{i=i_1}^{N} x_i = 1, \quad x_i \ge 0\,, \; N \ge 1 \tag{4}$$

In this case the frequency of *i*-th statistical cell within *i*-th fraction is equal to

$$\alpha_i = \frac{x_i}{C_i} \tag{5}$$

Where, α_i is the frequency of the *i*-th statistical cell in *i*-th fraction; C_i is the number of elementary cells in *i*-th fraction. Inserting Eq. (5) into Eq. (2) gives a specific entropy value (s) of each fraction *i*, then we can sum the specific entropy for each fraction and get the grading entropy of the soil:

$$S = -\frac{1}{\ln 2} \sum_{i=i_1}^{N} x_i \log_2 x_i + \sum_{i=i_1}^{N} x_i \log_2 C_i \tag{6}$$

Eq. (6) can be spilt into two parts

$$S = \Delta S + S_0 \tag{7}$$

Where ΔS and S_0 are the entropy coordinates, ΔS is called entropy increment, S_0 is called base entropy, and they can be expressed as follows:

$$\Delta S = -\sum_{i=i_1}^{N} x_i \log_2 x_i \tag{8}$$

$$S_0 = \sum_{i=i_1}^{N} x_i \log_2 C_i \tag{9}$$

In order to study the character of the grading entropy on relative levels, the two component coordinates (ΔS and S_0) of grading entropy may be normalized as follows (Lőrincz 1986):

$$A = \frac{S_0 - S_{0\min}}{S_{0\max} - S_{0\min}} = \frac{\sum_{i=1}^{N} x_i (i-1)}{N-1} \tag{10}$$

$$B = \frac{\Delta S}{\ln N} \tag{11}$$

Where, *A* and *B* are termed as relative base entropy and relative entropy increment, respectively; $S_{0\min}$ and $S_{0\max}$ are eigen-entropy of the smallest and largest fractions in the mixture.

3. MAXIMUM GRADING ENTROPY OF SOIL

Three series of optimal soil grading curves with maximum entropy based on the principle of maximum entropy were calculated and their particle size distribution curves were analyzed.

According to the principle of maximum entropy, the maximum *S* can be achieved using Lagrange multipliers as follows (Lőrincz et al, 2015; Singh, 2014):

$$L_{\max S} = -\sum_{i=1}^{N} C_i \frac{x_i}{C_i}\log_2\frac{x_i}{C_i} + \lambda\sum_{i=1}^{N}(x_i - 1) \qquad (12)$$

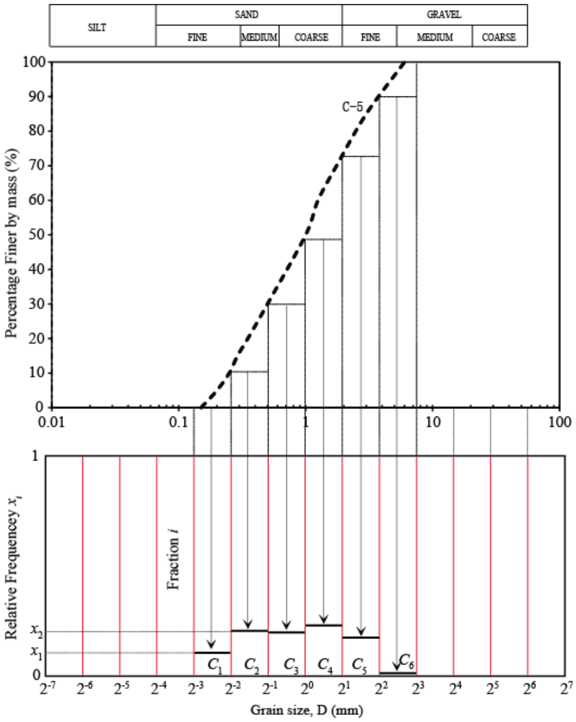

Figure 1 Discretization of particle size distributions to grading entropy cell system.

Where: $L_{\mathrm{Max}\,S}$ is the Lagrange function of maximum *S*; $\lambda\sum_{i=1}^{N}(x_i - 1)$ is the Lagrange multiplier and corresponding constraint. Differentiate the Lagrange function with respect to the relative frequencies, x_i, and equate the derivative to zero, we can obtain

$$\frac{x_i}{C_i} = 2^{\lambda - \frac{1}{\ln 2}} = \mathrm{cons.} \qquad (13)$$

According to Equation (13), we can easily find that the optimal PSD curve in this condition is a straight line when S=$S_{\max}$, and its slope will decrease with the increasing *N*. (See Figure 2). Now, following the same procedure as maximum *S*, the following equation can be obtained for the case of maximum Δ*S* :

$$x_1 = x_2 = x_3 = \cdots = x_N = 1/N \qquad (14)$$

From equation (14), it is easily to find that PSD curve is a straight line in the semi-logarithmic coordinate system (log *d* –*P*), when $\Delta S = \Delta S_{\mathrm{Max}}$. This means the optimal PSD curve in this condition is log linear when $\Delta S = \Delta S_{\mathrm{Max}}$, and its slope decreases with the increasing of *N*. (See Figure 3). Similarly, the following two equations can be obtained for the case of maximum B:

$$\Delta S_{\mathrm{Max}\,B} = -\left[(N-1)a^N - \frac{a^N-1}{a-1} + 1\right]\frac{a-1}{a^N-1} + \log_2\frac{(a^N-1)}{a-1} \qquad (15)$$

$$A_{\mathrm{Max}\,B} = \frac{1}{N-1}\left[\frac{N-1}{a^N-1}a^N - \frac{1}{a-1} + \frac{1}{a^N-1}\right] \qquad (16)$$

From equations (15) and (16), we can plot Figure 4 to interpret the relationship among maximum *B*, maximum ΔS, and maximum *A*. Figure 4 shows the maximum *S* and maximum ΔS are special cases of maximum *B*, with the increase of grading parameter *a*, the distributions of maximum *B* and maximum ΔS illustrate the same tendency. Maximum *S* is also a special case of maximum ΔS, there are two points of maximum *S* in the distribution of maximum ΔS and maximum *B*, compared to only one point of maximum ΔS in the distribution of maximum *B*.

Combining normalized entropy coordinates *A* and *B*, the relationship between *A* and *B* can be shown in Figure 5a, as well as its corresponding PSD curve (Figure 5b). Figure 5a shows the distribution of *A*-*B* is a semi-ellipse.

Given that the cases of maximum *S* and maximum ΔS are special cases of maximum *B*, so the grading curves in the condition of maximum grading entropy *B* are general optimal grading distributions. This means all the particle size distribution can be described in *A*-*B* space, wherein the internally unstable soils would plot inside some area within *A*-*B* space.

4. SOIL STABILITY ASSESSMENT AND VALIDATION OF PROPOSED MODEL

Based on above analysis, a new assessment method for evaluating the potential of internal instability of granular soils is proposed based on grading entropy.

1. Calculate the grading entropy using equations (3)-(11) based on discretization PSD (Fig. 1), get *A* and *B*.

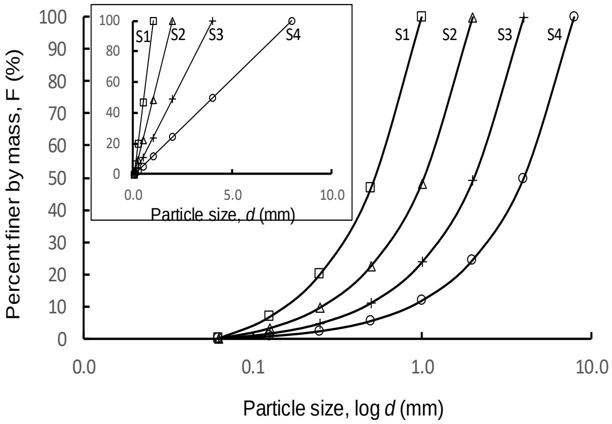

Figure 2 Optimal PSD Curve When S=$S_{\max}$.

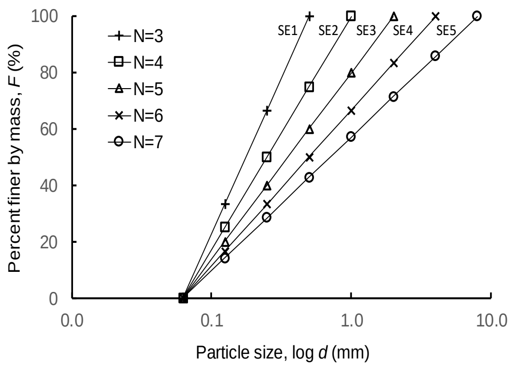

Figure 3 Optimal PSD Curve When Δ*S*=Δ*S*$_{max}$.

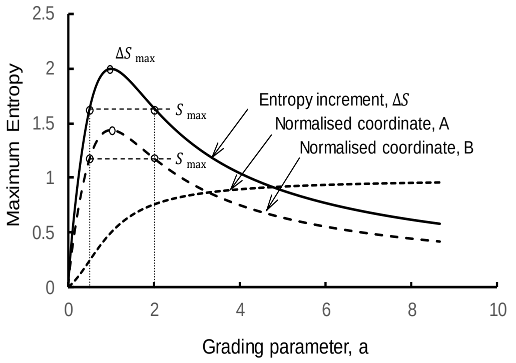

Figure 4 Distribution of Normalized Entropy Coordinates *A* and *B* and Entropy Increment Δ*S* with Increasing of Grading Parameter *a* when *B*=*B*$_{\max}$

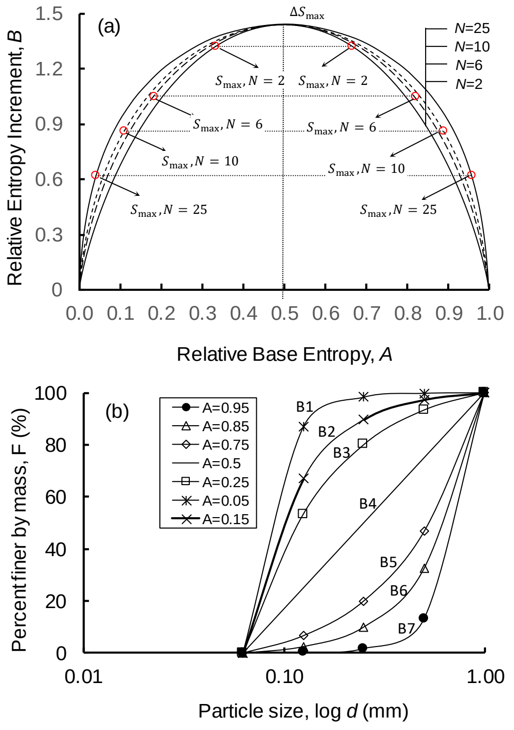

Figure 5 The Relationship Between Two Normalised Coordinate (a) and (b) Corresponding Optimal Particle Size Distribution Curve When N=4

2. Revise *B* with a correction coefficient, get an updated B' as follows:

$$B' = B \times \log_N^{C_u} \qquad (17)$$

3. Connect point (0,0) with the vertex of *A*-*B* curve and the vertex with point (1,0) (See Fig. 5a), and get following equation:

$$B = \begin{cases} \frac{2}{\ln(2)} A, & 0 \le A < 0.5 \\ \frac{2}{\ln(2)} - \frac{2}{\ln(2)} A, & 0.5 \le A \le 1 \end{cases} \qquad (18)$$

4. The soil can be finally assessed as have a potential of internal instability when the random $B' \le B$.

In order to show the application and validation of proposed model, a laboratory dataset of 46 samples is evaluated using the proposed model and two commonly used geometric criterion (i.e. Kenney and Lua 1985; and Kezdi 1979), as shown in Table 1. These soils contain uniformly-grade, gap-graded, well-graded and broadly-graded soils with their uniformity coefficients varying between 1 and 136.

From Table 1, there are 4 and 10 inconsistent predictions from Kenney and Lau's and Kezdi's criterion, and 3 inconsistent predictions from the current model. Notably, these 3 conservative assessments from the current model include 2 samples from Aberg (1991) and 1 from Indraratna et al. (2015). Nevertheless, all three criterion conservatively identify sample G of Aberg (1991) as internally unstable. However, rest of the conservative assessments from current model are not the same from the other two criterions. This implies that the proposed model differs significantly from the criterion of both Kenney and Lau (1985) and Kezdi (1979) and is more conservative. This is mainly because the existing criterions are merely based on the slope of the particle size distribution (PSD) curve (Chapius 1992), whereas the proposed model is based on the grading entropy, which accounts for the complexity and the uncertainty of soil grading.

Although the potential of internal instability is governed by particle size distribution, compressibility and other decisive factors; the proposed geometrical model is solely based on grading entropy. Nevertheless, while there are some inconsistent predictions, the current model can predict the potential of the internal instability more accurately as well as safely compared to the two existing criterions.

Based on the above analysis, it is found that the proposed entropy based model could be used to assess the potential of internal instability for published test data with more accuracy. The advantage of this method is that it can be applied easily for the prompt assessment of internal instability potential, for instance, two normalized entropy coordinates can be easily calculated from the particle size distribution curve, then using equations (16 to 18) or Figure 5a, the potential of internal instability of soils can be evaluated.

5. CONCLUSIONS

Three series of optimal particle grading curves based on grading entropy, principle of maximum entropy and the corresponding particle size distributions were analyzed. A new prompt geometrical criterion for assessing potential of internal instability was proposed based on principle of maximum entropy and the following conclusions were drawn:

- There are three conditions of maximum grading entropy based on the theory of grading entropy such as maximum *S*, maximum ΔS, and maximum *B* related to three series of optimal particle grading curves.
- The optimal grading curves in condition of maximum *B* are general optimal grading distributions, with the condition of maximum ΔS as the vertex of the curve, and the condition of maximum *S* on the curve. So the optimal grading curves in condition of maximum *B* can be commonly used.
- An assessment criterion of the potential of internal instability of soil based on the principle of maximum entropy was proposed, which mainly using two of normalized coordinates of grading entropy to evaluate the internal stability of soils. The new criterion accounted for the complexity and the uncertainty of soil grading from a new perspective, and it can be simply used.
- The analysis showed that the proposed entropy based criterion for assessing the potential of internal instability proved to be more accurate and safe compared to many existing criterion.
- The proposed method can be used for granular soils (e.g. granular filters for dams and railway subgrade) with allowable d_{100} greater than 96 mm, as illustrated for the existing test data.

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REFERENCES

Aberg, B. 1992. Void ratio of noncohesive soils and similar materials. *Journal of Geotechnical and Geoenvironmental Engineering*, 118: 1315-1334. (DOI: 10.1061/(ASCE)0733-9410(1992)118:9(1315))

Burenkova, V. V. 1993. Assessment of suffusion in noncohesive and graded soils. *Proc.*, *1st Int. Conf. Geo-Filters*, Karlsruhe, Germany, Balkema, Rotterdam, The Netherlands, 357-360.

Chang, D.S. & Zhang, L.M. 2013. Extended internal stability criteria for soils under seepage. *Soils and Foundations* 53(4):569-583.

Fannin, R., & Moffat, R. 2006. Observations on internal stability of cohesionless soils. *Géotechnique* 56(7): 497-500.

Honjo, Y., Haque, M. A., and Tsai, K. A. (1996). Self-filtration behavior of broadly and gap-graded cohesionless soils. *Geofilters'96*, Lafleur J. and Rollin A., eds., Bitech Publications, Montréal, 227-236.

Imre, E., Lőrincz, J., Trang,Q., P., et al. 2009. A general dry density law for sands. *KSCE Journal of Civil Engineering*, 13(4): 257-272.

Indraratna, B., Nguyen, V. T., & Rujikiatkamjorn, C. 2011. Assessing the potential of internal erosion and suffusion of granular soils. *Journal of Geotechnical and Geoenvironmental Engineering* 137(5): 550-554 (DOI: 10.1061/(ASCE)GT.1943-5606.0000447)

Indraratna, B., Nguyen, V. T., & Rujikiatkamjorn, C. 2012. Hydraulic conductivity of saturated granular soils determined using a constriction based technique. *Can. Geotech. J.*, 49(5): 607–613.

Indraratna, B., Israr, J., & Rujikiatkamjorn, C. 2015. Geometrical method for evaluating the internal instability of granular filters based on constriction size distribution. *Journal of Geotechnical and Geoenvironmental Engineering* 141(10): 04015045 (DOI: 10.1061/(ASCE)GT.1943-5606.0001343).

Israr, J., Indraratna, B., & Rujikiatkamjorn, C. 2016. Laboratory modelling of the seepage induced response of granular soils under static and cyclic conditions. *Geotechnical Testing Journal* 39(5): 1-18 (DOI: 10.1520/GTJ20150288)

Israr, J. 2016. Internal instability of granular filters under cyclic loading. Ph.D. thesis, Univ. of Wollongong, Wollongong City, Australia.

Israr, J., & Indraratna, B. 2017. Internal stability of granular filters under static and cyclic loading. Journal of Geotechnical and Geoenvironmental Engineering: 04017012, 1-16, 143(6) (https://doi.org/10.1061/(ASCE)GT.1943-5606.0001661).

Kenney, T. C., & Lau, D. 1985. Internal stability of granular filters. *Canadian Geotechnical Journal* 22: 215–225.

Kezdi, A. 1979. Soil physics, Elsevier Scientific, Amsterdam, The Netherlands.

Lafleur, J., Mlynarek, J., & Rollin, A. 1989. Filtration of broadly graded cohesionless soils. Journal of Geotechnical and Geoenvironmental Engineering 115(12): 1747–1768. (DOI: 10.1061/(ASCE)0733-9410(1989)115:12(1747))

Li, M. 2008. Seepage induced instability in widely graded soils. Ph.D. thesis, The University of British Colunmbia, Vancouver City, Canada.

Lőrincz, J. 1986. Grading Entropy of Soils. Ph. D. Thesis, *Technical University of Budapest*, Budapest, Hungary. (In Hungarian)

Lőrincz, J. 1990. Relationship between grading entropy and dry bulk density of granular soils. *Period. Polytech.*, Chem. Eng., 34(3):255–265.

Lőrincz, J., Imre, E., Gálos, M., et al. 2005. Grading entropy variation due to soil crushing. *International Journal of Geomechanics*, 5(4):311-319.

Lőrincz, J., Imre, E., Fityus, S., et al. 2015. *Entropy*, 17: 2781-2811.

Moffat, R., & Fannin, R. J. 2006. A large permeameter for study of internal stability in cohesionless soils. *Geotechnical Testing Journal* 29(4): 273-279.

Nguyen, V. T., Rujikiatkamjorn, C., & Indraratna, B. 2013. Analytical solutions for filtration process based on constriction size concept. *Journal of Geotechnical and Geoenvironmental Engineering* 139(7): 1049–1061. (DOI: 10.1061/(ASCE)GT.1943-5606.0000848)

Raut, A. K., and Indraratna, B. (2008). Further advancement in filtration criteria through constriction-based techniques. *Journal of Geotechnical and Geoenvironmental Engineering* 134(6): 883-887 (DOI: 10.1061/(ASCE)1090-0241(2008)134:6(883)).

Singh V P. 2014. Entropy theory in hydraulic engineering: an introduction. *American Society of Civil Engineers*, ProQuest Ebook Central.

Sherard, J. L. 1979. Sinkholes in dams of coarse broadly graded soils. *Proceedings of 13th Congress on Large Dams*, New Delhi, 2: 25-35.

Skempton, A. W., & Brogan, J. M. 1994. Experiments on piping in sandy gravels. *Géotechnique* 44(3): 449–460.

Terzaghi, K. 1939. Soil mechanics—A new chapter in engineering science. *Journal of Institution of Civil Engineers* (UK), 12(7): 106–141.

Unites States Army Corps of Engineers (USACE). 1953. Investigation of filter requirements for underdrains. *Technical Memorandum.* No. 3–360, U.S. Waterways Experiment Station, Vicksburg, Mississippi.

Wan, C. F., and Fell, R. 2008. Assessing the potential of internal instability and suffusion in embankment dams and their foundations. *Journal of Geotechnical and Geoenvironmental Engineering* 134(3): 401–407.