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PROBABILISTIC METHODS FOR QUANTIFICATION AND MAPPING OF GEOHAZARDS

Farrokh Nadim, International Centre for Geohazards, Norwegian Geotechnical Institute, Oslo, Norway
Suzanne Lacasse, International Centre for Geohazards, Norwegian Geotechnical Institute, Oslo, Norway

Abstract

The basic understanding of geohazards and the ability to deal with the risks involved necessitate increased use of probabilistic methods, as they provide a rational framework that account for uncertainties. Probabilistic methods can be applied to slope stability evaluation and estimation of the annual probability of slope failure, which are essential elements in any geohazard study. This paper reviews the different steps of geohazard assessment, the probabilistic models for slope stability evaluation, and methods for estimating the annual probability of slope failure. Furthermore, a consistent framework for landslide hazard zonation or mapping is put forward.

Résumé

Afin de bien comprendre l'importance des géohazards et d'améliorer la perception des risques associés, il est nécessaire d'utiliser de plus en plus les méthodes probabilistes, car celles-ci offrent un cadre rationnel qui prend en compte les incertitudes. Les méthodes probabilistes pouvent être appliquées à la fois pour evaluer la stabilité des pentes et estimer la probabilité annuelle de rupture, deux éléments essentiels à une étude des géohazards. L'article fait une revue des différentes étapes d'une étude des géohazards, des modèles probabilistes pour l'étude de la stabilité des pentes et des méthodes pour estimer la probablité annuelle de rupture. Une méthodologie pour le zonage des hazards de glissement et pour la cartographie du risque est proposée.

1. INTRODUCTION

The need to improve the basic understanding of geohazards and the ability to deal with the risks involved necessitate increased use of probabilistic methods, as they provide a rational framework that account for uncertainties. Application of probabilistic methods to slope stability evaluation and estimating the annual probability of slope failure are essential elements in most geohazards studies.

Establishing a model of the slide frequency (i.e. the annual probability of failure) is required in order to perform risk evaluation. When the potential triggering source of a slide is clearly identified, for example a strong earthquake in a seismically active region, then estimating the annual probability of slope instability is theoretically straight-forward. However, when the potential triggering mechanism is not obvious and the slope stability calculations essentially provide an estimate of failure probability for static conditions, then estimating the annual failure probability is not straightforward. In this latter situation, geological evidence and dating of previous slides are the key parameters for estimating the annual failure probability. Both situations are addressed here and ideas for a consistent framework for hazard zonation mapping are put forward.

2. GEOHAZARDS AND THEIR ASSESSMENT

Today, society and regulations require that the risk of civil engineering structures and infrastructure be quantified. Risk cannot be evaluated without involving multi-disciplinary parameters. In addition, political aspects and public opinion need to be considered. Statistics, reliability analyses and risk estimates are useful tools that assist the decision-making in terms of hazards affecting a population.

Geological risks, or "geohazards", are events resulting from geological features and processes that present severe threats to humans, property and the natural and built environment. Landslides caused by heavy rainfall, floods, earthquakes, erosion, and human activities are the most common geohazards on land. Near-shore and off-shore, various geological processes, earthquakes and human activities, for instance in connection with petroleum exploration and production, can also trigger slides and large mass flows.

The urgent need to improve the understanding of geohazards and the ability to deal with the risks has generated significant research and development activity in this field during the past decade. The need is accentuated by increased sliding and flooding in many regions, increased concern for geohazards in production and transport of oil and gas and increased vulnerability to earthquakes. Facts supporting this urgency include:

- The 1999 World Disaster Report estimated that in the period 1988-1997 landslides alone caused

9,000 deaths and total damages of about USD 450 million.

- In 1999, the extreme rainfall in Venezuela triggered severe flooding and landslides, and alone caused over 20,000 deaths.
- The largest oil companies define reduction of risk due to geohazards in deep water as one of their top research priorities. The consequences of accidents due to geohazards offshore in terms of loss of life and damage to the environment would be catastrophic.
- Tsunamis (large waves formed by rapid mass movements) present extreme threats to coastal areas. In the 1990s, four tsunamis ravaged Nicaragua, Indonesia, Japan, and Papua New Guinea causing the loss of 4,000 lives.
- Recent earthquakes in Turkey (1999), El Salvador (2001) and India (2001) caused loss of 40,000 lives and made many more homeless. The large 2001 earthquake in Seattle caused much damage but no loss of life, due to mitigation efforts in the USA. The Munich Re insurance company assess losses for the last 5 decades to be respectively USD 1, 21, 55, 82, and 219 billion (2000 equivalents). Escalation is likely, with incre vulnerability.
- In 2001, natural hazards caused over 25,000 deaths, or twice the number of lives lost in the year 2000. Material damages in 2001 alone amount to USD 300 billion according to Munich Re.

Climate research indicates that one can expect more extreme weather in the next 50 years, leading to increased landslide and vulnerability. Predicting the hazard posed by geological processes, and evaluating the human, environmental and economical consequences of geohazards require that the uncertainties in different parts of the problem are quantified and addressed properly. An integrated scientific approach involving many disciplines is required for this purpose.

Figure 1 shows the interdisciplinary nature of a typical geohazard study. As seen on the figure, assessment of geohazards and their associated risks requires identification and analysis of the possible failure scenarios (failure modes, triggers and related failure consequences) that can give a significant contribution to the total risk. Furthermore, the annual probability that the potential triggering mechanism is strong enough to cause failure needs to be evaluated.

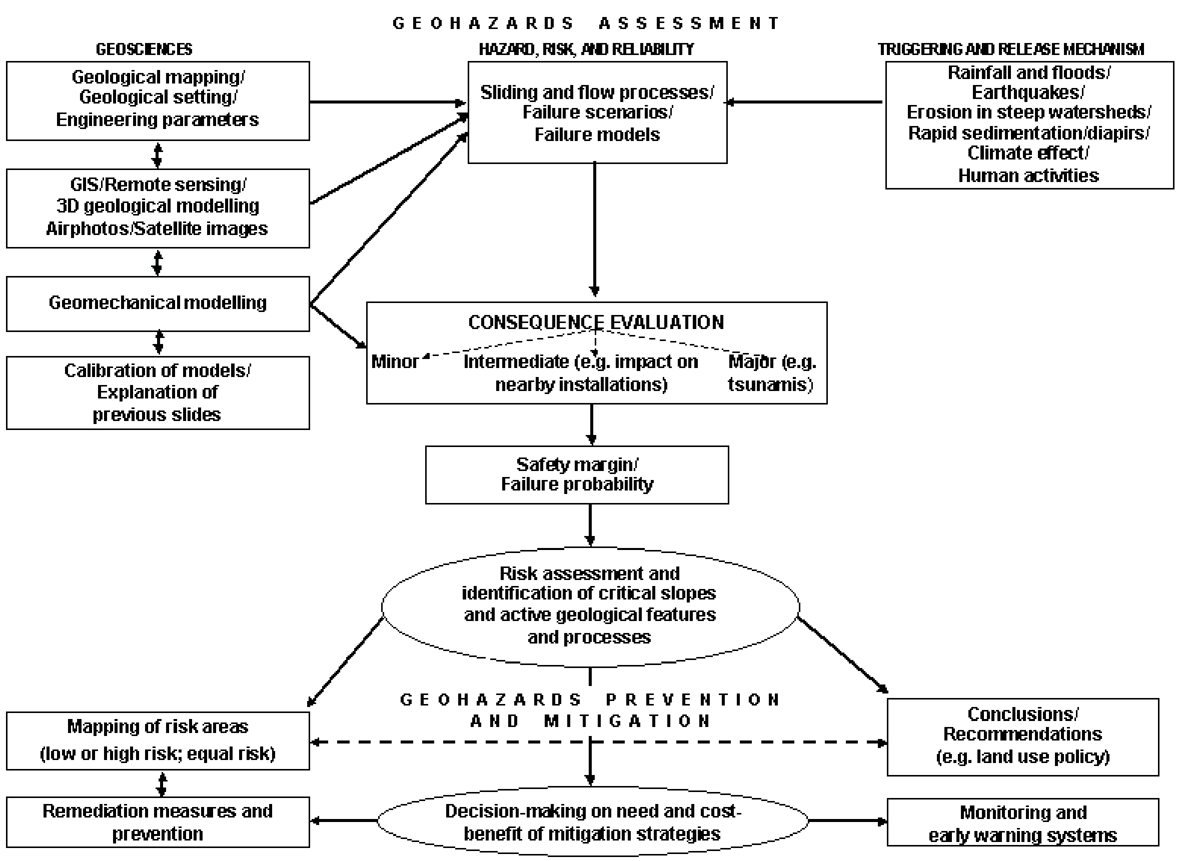

Figure 1. Interdisciplinary assessment of geohazards.

3. TYPES AND SOURCES OF UNCERTAINTY

There are significant uncertainties in all the steps of any geohazard evaluation. These are related to the assessment of soil and/or rock properties and in situ stresses, pore pressure and temperature conditions, identification and quantification of the triggers, definition the relevant failure modes, and the estimation of the run-out distance and other consequences of the slide.

It is conceptually useful to classify the uncertainty in the mechanical soil/rock properties and load effects (e.g. design seismic acceleration coefficient) into two groups:

- Aleatory uncertainty represents the natural randomness of a variable. For example, the variation of the soil characteristics in the lateral direction is aleatory; the variation in the peak acceleration of an earthquake is aleatory. The aleatory uncertainty is also called the inherent uncertainty. Aleatory uncertainty cannot be reduced.

- Epistemic uncertainty represents the uncertainty due to lack of knowledge on a variable. Epistemic uncertainty includes measurement uncertainty, statistical uncertainty (due to limited information), and model uncertainty. Epistemic uncertainty can be reduced, for example by increasing the number of tests or by improving the measurement method.

Within a geological unit, the mechanical properties are affected by both aleatory and epistemic uncertainties. In some locations, the aleatory uncertainty is very small (e.g. the soil exhibits very little spatial variation of properties) and most of the uncertainty is due to lack of knowledge. In other locations, the natural scatter in the material properties is large and aleatory uncertainty is more important than epistemic uncertainty. Unfortunately it is not possible to establish a set of guidelines for the evaluation of the uncertainty in soil and rock properties that are valid for all sites.

The epistemic uncertainty can be statistical, measurement-related and/or model-related. Statistical uncertainty is due to lack of information such as limited number of observations. Measurement uncertainty is due to, for example, imperfections of an instrument or of a method to register a quantity. Model uncertainty is due to idealizations made in the physical formulation of the problem.

Statistical uncertainty is present because the parameters are estimated from a limited set of data, and is affected by the type of estimation technique used. Measurement uncertainty is described in terms of accuracy and is affected by bias (systematic error) and by precision (random error). It can be evaluated from data provided by the manufacturer, laboratory tests and/or scaled tests. Model uncertainty is defined as the ratio of the actual quantity to the quantity predicted by a model.

When determining the uncertainties in a material property, it is important to ensure that the data are consistent with the geological interpretation for the site, and that consistent data populations are used (Lacasse and Nadim, 1996). Important uncertainties have been introduced in the past because of inconsistent data sets. The inconsistency can originate from different soil and rock types, different stress conditions, different test methods, stress history, different codes of practice, testing errors or imprecision that are not reported, different interpretations of the data, sampling disturbance, etc.

4. TRIGGERING MECHANISMS FOR SLIDES

The triggers for different slope failures can be natural, on-going processes or human activities. A distinction can be made between stress (or load) increasing triggers bringing the stress conditions in the soil mass closer to failure, and strength decreasing triggers causing strength loss due to large strains and pore pressure changes. The most common triggers for onshore and offshore slope failures include:

- Human activities, in particular construction activities related to roads, tunnels, bridges, etc.
- Extreme rainfall.
- Earthquake activity causing short-term inertia forces and post-earthquake pore pressure increase and fault displacements.
- Rapid deposition leading to excess pore pressure conditions, underconsolidation and increased shear stress level in a slope.
- Toe erosion or top deposition giving higher slope inclination and increased gravity forces and shear stress along potential failure surfaces.
- Sensitive (contractive) and collapsible soils, which could lead to retrogressive sliding and increased spatial extent of failure zones.

Probabilistic analyses of slope stability and quantitative assessment of geohazards require that the relevant triggering mechanisms and their induced load effects be described in probabilistic terms.

5. PROBABILISTIC SLOPE STABILITY EVALUATION

Slope instability is the most common and serious type of geohazard. In its simplest form, the trigger is gravity and the calculation model is usually some sort of a limit equilibrium analysis. Even in this situation, where there is virtually no uncertainty in the intensity of the trigger, there are uncertain factors that affect the safety margin of a slope.

The first-order, second-moment (FOSM) approach provides approximations for the mean and standard deviation of safety margin only. The FOSM approximation suffers from a lack of invariance, where different, but equivalent, definitions of safety margin may lead to different estimates of failure probability.

Monte Carlo simulation is another powerful technique that may be used to estimate the probability of slope instability. The Monte Carlo simulation is implemented in some commercial slope stability analysis packages. However, when the probability of failure is very small, the procedure is very time-consuming.

Nadim and Lacasse (1999) described a probabilistic slope stability analysis based on the first- and second-order reliability methods (FORM and SORM, Hasofer and Lind 1974) and the generalized method of slices. In FORM or SORM, a performance function g(X), is defined such that $g(X) \geq 0$ means that the slope is stable and $g(X) < 0$ means that the slope has failed. X is a vector of basic random variables including soil properties, load effects, geometry parameters and modeling uncertainty. If the joint probability density function $F_x(X)$ is known, then the probability of failure P_f is given by

$$P_f = \int_L F_x(X)dX \qquad [1]$$

where L is the domain of X where $g(X) < 0$.

In general the above integral cannot be solved analytically, and an approximation is obtained by the FORM approach. In this approach, the general case is approximated to an ideal situation where X is a vector of independent Gaussian variables with zero mean and unit standard deviation, and where g(X) is a linear function. The probability of failure P_f is then:

$$P_f = P\,(g(X) < 0) = P\,(\sum_{i=1}^{n} \alpha_i X_i - \beta < 0) = \Phi\,(-\beta) \qquad [2]$$

where α_i is the direction cosine of random variable X_i, β is the distance between the origin and the hyperplane $g(X) = 0$, n is the number of basic random variables X, and Φ is the standard normal distribution function.

The vector of the direction cosines of the random variables (α_i) is called the vector of sensitivity factors, and the distance β is called the reliability index.

The higher the reliability index β computed from Eq. 2, the smaller the probability of failure. The square of the direction cosines or sensitivity factors (α_i^2), whose sum is equal to unity, quantifies in a relative manner the contribution of the uncertainty in each random variable X_i to the total uncertainty.

Nadim and Lacasse (1999) presented an example of probabilistic slope stability evaluation with FORM, see Figure 2. The deterministic analysis gave a safety factor (FS) of 2.10 for slip surface 1 (critical slip surface from deterministic analysis) and 2.67 for slip surface 2. The probability of failure was however 8 times higher for slip surface 2 than for slip surface 1. In other words, the analysis indicated that the slip surface with the lowest deterministic safety factor (FS = 2.10) is 8 times less likely to fail than the critical probabilistic slip surface with a higher safety factor (FS = 2.67). The inconsistency is due to the effects of the uncertainties in the input parameters in the analyses.

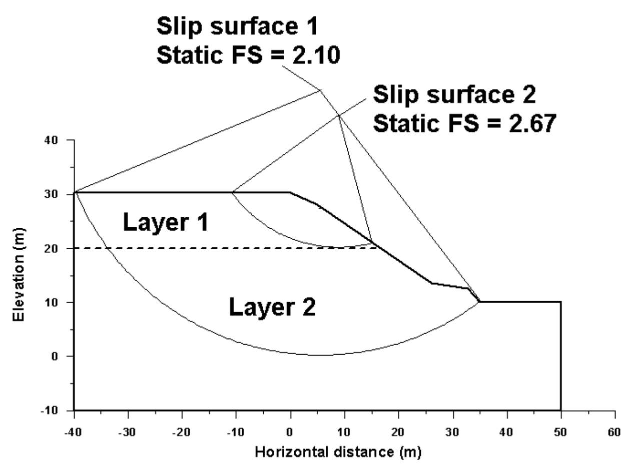

Figure 2. Example slope considered by Nadim and Lacasse (1999)

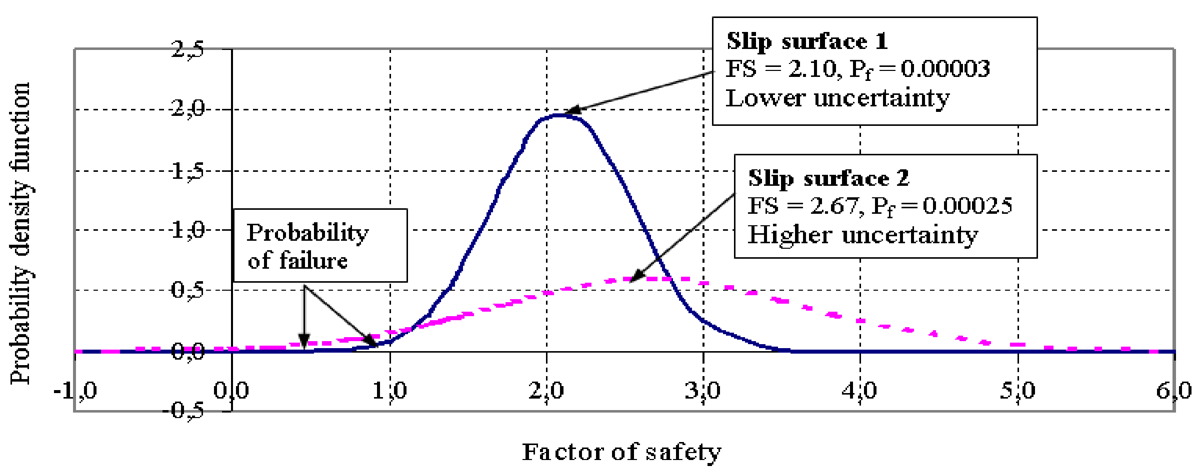

Figure 3. Distribution of safety factors for 2 slip surfaces.

This example clearly shows that the factor of safety is not a sufficient indicator of safety margin because the uncertainties in the analysis parameters affect the probability of failure, while they do not intervene in the conventional calculation of safety factor. The flatter distribution of the factor of safety in Figure 3 reflects greater uncertainty and will result in a greater probability of failure (area under the curve where FS < 1).

6. ESTIMATION OF THE ANNUAL PROBABILITY OF FAILURE

The annual probability of slope failure may be estimated from the geological evidence, e.g. observed slide frequency, geological history, geophysical investigations, and radiocarbon dating of sediments; and/or analytical simulations like the FORM/SORM approach mentioned above. Ideally, both approaches should be employed.

If the trigger for inducing a slide is identified, the annual probability of slope instability can be established by evaluating the conditional probability of failure for different return periods of the trigger. The conditional probabilities are then integrated over all return periods to obtain the unconditional failure probability. Calculation can be simplified by using the approximation suggested by Cornell (1996) or a similar approach.

When the triggering mechanism is not obvious, the probabilistic slope stability calculations provide an estimate of failure probability for static conditions. In this situation, it is not straightforward to relate the calculated "timeless" failure probability to a failure frequency. Nadim (2002) and Nadim et al. (2003) developed several ideas for quantifying the annual probability of slope instability:

- Bayesian approach with Bernoulli sequence
- Statistical model for failure frequency
- Availability problem - Markov chain
- Interpretation of the static failure probability as the instantaneous hazard function
- Bayesian interpretation of static failure probability

The first two approaches are purely statistical and do not involve any geotechnical calculations. Their input is the frequency of slide events (or lack thereof), which may be based on observations or inferred from geological evidence, for example dating of slide sediments. The third approach combines the calculated probability of static slope failure with the slide frequency estimated from the geological evidence. The last two approaches are mainly based on the calculated probability of static failure.

6.1 Bayesian approach with Bernoulli sequence

The Bayesian approach is a powerful method (e.g. Folayan et al. 1970). The annual probability of slide release could be anywhere between zero and one if no information is available about the slope and there are no recorded observations (diffuse prior). If, after "n" years of observation, sliding activity is observed in "r" years, then the (posterior) annual probability of slide occurrence can be obtained by the Bayesian approach (Ang and Tang 1984). This model considers the annual failure of a given slope as a Bernoulli sequence. The status of the slope during a one-year interval is seen as a trial with two possible outcomes: failure or non-failure. The approach makes the following assumptions:

1. Each trial has only two possible outcomes: the occurrence or non-occurrence of sliding.
2. The probability of occurrence of sliding in each trial is constant.
3. The trials are statistically independent.

Assumptions 1 and 3 are reasonable, but the validity of assumption 2 is arguable.

Figure 4 shows the distribution of annual failure probability estimated by this approach for slopes having not failed during 1, 3, and 8 years of observation.

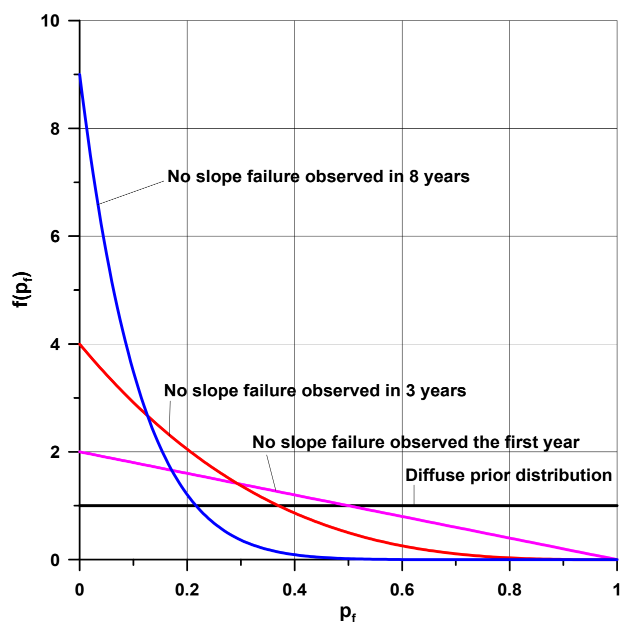

Figure 4 Distribution of annual failure probability of a slope having not failed during several years of observation.

6.2 Statistical model for failure frequency

This statistical model considers the sliding activity as a Poisson process, with the following assumptions (Ang and Tang 1984):

1. Sliding can occur at random and at any time or any point in space.
2. The occurrence(s) of a slide in a given time (or space) interval is independent of that in any other non-overlapping intervals.
3. The probability of slide occurrence in a small time interval is proportional to the time interval,

the mean rate of slide occurrence is constant and the probability of two or more occurrences in the time interval is negligible.

The number of slide occurrences in a time interval is given by the Poisson distribution. In this model, assumptions 2 and 3 are believed to be justifiable, but assumption 1 is an oversimplification.

On the basis of these assumptions, the number of slide occurrences in the time interval t is given by the Poisson distribution. The mean occurrence rate for the Poisson process could be estimated from dating of slide sediments, or other geological evidence.

6.3 Availability problem – Markov chain model

The assumption of stochastic independence is a fundamental assumption in the above Bernoulli trials and Poisson-process models. In many applications, however, significant dependence exists between the successive trials or years or steps that can be identified in a physical process. This means that the transition of a system from one state to another may generally depend on the prior states of the system. If the transition probability depends only on the current state, the process of change is called a Markov chain.

Nadim (2002) presents a model for combining the “timeless” probability of failure computed from a probabilistic slope stability evaluation with the slide frequency estimated from geological evidence to obtain a better estimate of the annual failure probability. The model considers the stability of the slope as an “availability problem” with two states: safe and failed. The availability problem is a classical application of the Markov chain model. In most typical situations, this model predicts an annual failure probability that is very close to the slide frequency estimated from geological evidence.

6.4 Interpretation of computed static failure probability as the instantaneous hazard function

The FORM analysis (or a similar approach) takes into account uncertainties within the system to provide an estimate of the probability that a system, if “built” today, would fail immediately. This interpretation is straight-forward for man-made slopes, for example dams and embankments. However, interpretation of the FORM results for stability of a natural slope is not straightforward. The slope of interest is standing today, so its probability of “static” failure is zero. Obviously, if there are no triggers or deterioration mechanisms present in the system, the slope that stands today would never fail.

Therefore, the probability of static failure provided by FORM analysis is not the annual probability of failure. However, the results of FORM analysis could still be used to estimate the annual failure probability. Two possible interpretations of FORM results that may be used to obtain the annual probability of failure are presented by Nadim et al. (2003).

A concept that is useful for the interpretation of FORM results and the estimation of annual P_f is the hazard function (Melchers 1999). The hazard function (also called ‘age specific failure rate’ or ‘conditional failure rate’) expresses the likelihood of failure in the time interval t to t+dt as dt approaches 0, given that failure has not occurred prior to time t:

$$h_T(t) = P\left(failure\ between\ t\ and\ t+dt \middle| no\ \ failure\ \ prior\ to\ t\right)$$

More rigorously,

$$h_T(t) = \frac{P(t \le T \le t+dt)}{1-P(T \le t)} = \frac{f_T(t)}{1-F_T(t)} \qquad [3]$$

One approach to estimate the annual failure probability is to interpret the failure probability computed with FORM as the cumulative distribution function value at the present time (the present being time “t” after the last failure). Thus, by assuming the shape of the hazard function, knowing (or estimating) the time of the last failure, and manipulating the hazard function equations, one can obtain the value of the hazard function at the present time. Since the hazard function is a conditional density function, its value at the present time is the probability density given that no failure has occurred up to this point. The annual probability of failure can be obtained by simply integrating the hazard function over the next year.

Nadim et al. (2003) applied this approach to a submarine slope in the Gulf of Mexico which had a computed static safety factor of about 1.5 and a computed probability of static failure of $P_f = 4.2 \cdot 10^{-4}$. Depending on assumptions made for the hazard function, the result was an annual failure probability in the range of 10^{-6} to 10^{-8}.

6.5 Interpretation of computed static failure probability in a Bayesian framework

An alternative interpretation of the failure probabilities computed with FORM is more consistent with the theory behind the FORM analysis, but it also requires a number of assumptions (Nadim et al. 2003)

The fact that the slope is standing today implies that the current factor of safety, although unknown, is greater than one. Thus, the question of annual probability of failure becomes the question of the likelihood that the current factor of safety will fall below one during next year. The current factor of safety is unknown, but its distribution can be computed (distribution from FORM analysis, but truncated to reflect the fact that the slope is stable today). This interpretation is basically a Bayesian updating procedure where the a-priori information is that FS ≥ 1. Formally, the updated (or posterior) distribution of the factor of safety is:

$$P[FS < z \mid FS \ge 1] = \frac{F_{FS}(z) - F_{FS}(1)}{1 - F_{FS}(1)} \qquad [4]$$

The slope will fail during the next year only if its current value of safety factor is such that, with the given rate of deterioration, it will fall below unity during one year. This very simple calculation can be performed in several slightly different ways. Using this approach for the same submarine slope mentioned in the previous section, Nadim et al. (2003) obtained annual failure probabilities in the range between 10^{-7} and 10^{-9} (depending on the assumptions made).

It is clear that additional research is needed to formalize the interpretation of the annual failure probability on the basis of the "timeless" failure probability obtained by FORM, SORM, or a similar method. However, the alternative interpretations presented in sections 6.4 and 6.5 both give annual failure probabilities that are several orders of magnitude lower than the "timeless" probability of failure computed by FORM.

7. APPLICATION OF PROBABILISTIC METHODS IN HAZARD ZONATION

In its simplest form, probabilistic landslide hazard zonation has three requirements:

1. The "safe" and "unsafe" areas should be defined in a probabilistic framework. For example, areas where the annual probability of being affected by a landslide is less than 10^{-3} may be considered safe, while areas with a greater annual probability may be considered unsafe.
2. A probabilistic/statistical model describing the annual probability of slide release.
3. A probabilistic/statistical model describing the run-out distance for the slides.

This paper has focused on the second requirement, i.e. probabilistic models for slide release. The third requirement is far more complicated and the subject of on-going research at the highest technical level.

An example of a probabilistic approach for slide and avalanche hazard zonation was presented by Harbitz et al. (2001). They used a mechanical probabilistic model for avalanche release in combination with a statistical-topographical model for avalanche run-out distance to obtain the unconditional probability of extreme run-out distance.

For the mechanical model, FORM and Monte Carlo simulations were compared for calculating the annual probability of avalanche release. The comparison showed that the FORM approximation gave results that were almost identical to the simulations.

Harbitz et al. also discussed the interpretation of the statistical/topographical model for slide run-out as an extreme value model vs. a single value model. The ambiguous interpretation of the model reflects the need for more than one observation in a sufficient number of avalanche paths. They outlined how a "safe" run-out angle could be calculated based on each of the two approaches, and how a specified certainty level can be found by constructing confidence intervals based on the annual probability of slide/avalanche release.

Example applications in hazard zoning were presented with emphasis on how the influence of historical observations, local climate, etc., on run-out distance can be quantified in statistical terms and how a specified certainty level can be found by constructing confidence intervals for e.g. the most likely largest run-out distance during different time intervals. Owing to the quantified uncertainty in the probability of extreme run-out distance, it is suggested to indicate the areas susceptible to avalanches by zones rather than demarcation lines only.

8. CONCLUSIONS

Probabilistic approaches are a necessary and useful complement to conventional engineering analyses, as they provide important additional information on the effects of uncertainty on the response. To improve the basic understanding of geohazards and the ability to deal with the risks requires increased use of probabilistic methods. Probabilistic analyses of slope stability and estimation of the annual probability of slope failure are essential elements in the assessment of geohazards. The first requirement in the application of reliability-based approach is a clear understanding of the mechanisms of the situation modeled and sound engineering judgment to help quantify the uncertainties.

An example of slope stability calculation showed that the deterministic critical slip surface is not necessarily the one with highest probability of failure. Factor of safety is not a sufficient indicator of safety margin because the uncertainties in the analysis parameters affect probability of failure, but not factor of safety.

This paper suggests approaches to obtain the annual probability of failure for slopes, both where the trigger of the slide is clearly identified and where the triggering mechanism is not obvious. Suggestions for a framework to for hazard zonation mapping are also put forward.

For slopes where the trigger for inducing a slide is identified, the annual probability of slope instability can be established by evaluating the conditional probability of failure for different return periods of the trigger. The conditional probabilities are then integrated over all return periods to obtain the unconditional failure probability.

For slopes where the triggering mechanism for inducing a slide is not well-defined or uncertain, geological evidence and dating of previous slides are key parameters for estimating the annual failure probability. Five approaches are presented in the paper to estimate the annual probability of failure. In practice, the authors recommend that, depending on the data available for input in each of the approaches, as many as of the approaches as possible be applied in order to delimit a likely range of probability of failure.

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