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ESTIMATING THE AVERAGE FRICTION COEFFICIENT IN A SIMPLIFIED SNOW AVALANCHE DYNAMICS MODEL FOR SHORT SLOPES IN CANADA

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ABSTRACT

Avalanche dynamics models may be used to approximate the velocity and runout distances for extreme avalanches. The friction coefficients for most avalanche dynamics models are estimated from measured extreme runouts in large avalanche paths, limiting their utility for shorter paths. To better estimate the sliding friction in a dynamics model for short slopes, data were collected at 48 paths in the Coast, Columbia and Rocky Mountains of western Canada and at several paths in Quebec, Canada. Field studies included topographic surveys and estimation of the extreme runout position in each path. The average friction coefficient for each path was determined from the extreme runout position using a simplified Leading Edge Model. Multiple regression was used to relate the average friction coefficient to two topographic variables that can be easily measured in the field.

RÉSUMÉ

Les modèles de la dynamique des avalanches peuvent être utilisé pour estimer la vitesse et les distances d'arrêt pour les avalanches extrêmes. Pour la plupart des modèles, les coefficients de friction sont estimés à partir de distances d'arrêt extrême mesuré dans des grands couloirs d'avalanches, limitant leur utilité pour les couloirs courts. Afin de mieux pouvoir estimer la friction pour les couloirs courts, des données ont étaient recueillis dans 48 couloirs d'avalanche dans les Montagnes Côtières, la chaîne Columbia et les Rocheuses du Canada occidental et de plusieurs couloirs au Québec. Des études sur le terrain comprenaient des relevés topographiques, ainsi que l'estimation de la distance d'arrêt extrême pour chaque couloir. En utilisant un modèle 'Leading Edge' simplifié, le coefficient de friction fut déterminé pour chaque couloir d'avalanche utilisant la position d'arrêt extrême. Une régression multiple fut utilisé pour établir un rapport entre le coefficient de friction et deux variables topographiques, qui peuvent être mesurer sur le terrain aisément.

1. INTRODUCTION

In Canada, snow avalanches affect people, backcountry recreation and developments including roads, residential areas, industrial facilities, mines, railways, power and communication transmission lines, ski resorts, and forestry operations. In areas where avalanche terrain and human activities overlap, it is often important to define where avalanches can occur and how far they will run on and near the bottom of a slope. The runout of an avalanche can be defined as the point of farthest reach of an avalanche deposit within an avalanche path (McClung and Schaerer, 1993, p. 115). Specification of the runout distance for the largest, or extreme, avalanche expected within a path is of great importance for land-use planning and zoning in snow avalanche prone areas. Specification of velocity within the runout zone is also an important part of risk-based avalanche hazard zoning methods and the design of avalanche defence structures. Additionally, there are important economic considerations involved when specifying runout distances, since certain types of land uses may be excluded due to zoning restrictions based on avalanche risk.

Extensive research on the estimation of runout distances has typically focused on slopes with fall heights greater than 300 m. Short slopes are believed to form a distinct runout population (McClung and Lied, 1987; Schaerer, 1991). Since 1950, avalanches have killed 31 people in and near residential or public buildings in Canada in six avalanche events (Stethem and Schaerer, 1979, p. 89-93; Stethem and Schaerer, 1980, p. 19-23; Schaerer, 1987, p. 14-15; Jamieson and Geldsetzer, 1996, p. 171-173, 178-179; Government of Quebec, 2000). Of this number, twenty fatalities (65%) occurred at the base of slopes with vertical fall heights of 150 m or less, illustrating the importance of the understanding of avalanche runout on short slopes.

This paper presents a simple method for estimating the friction coefficient in a simplified Leading Edge avalanche dynamics model (McClung and Mears, 1995). Multiple regression is used to relate the average friction coefficient to easily measured topographic parameters. Data from four Canadian mountain ranges are used in the analyses.

2. BACKGROUND

The best methods for determining runout distances use direct evidence, including long-term observations of avalanches, observations of damage to vegetation, ground or structures, historical records from air photographs, newspapers, or oral communications (McClung and Schaerer, 1993, p. 115). The use of these methods may be limited where the historical record of avalanches is not sufficiently long, areas in the runout zone have been damaged by human activity such as logging, or there is no vegetation in the runout zone (e.g. alpine areas and northern latitudes). In these cases, and when a better understanding of avalanche velocity and impact pressure is required, models are typically used. Currently, two modelling approaches are used to predict extreme avalanche runout distances: the statistical method and conventional (dynamics) method (McClung and Schaerer, 1993, p. 115). The statistical method is a statistical evaluation of historical avalanche runouts in a given mountain range, applied to a given avalanche path (e.g. Lied and Bakkehøi, 1980; McClung and Mears, 1991; McKittrick and Brown, 1993). Jones (2002) and Jones and Jamieson (2003) have developed statistical models for predicting runout distances for short slopes in Canada using multiple regression and the runout ratio method. The main disadvantages of statistical methods are that they do not work for atypical paths or paths that run-up the opposite side of the valley.

The avalanche dynamics method involves estimating friction coefficients to calculate the velocity of an avalanche along an incline, and then defining the runout of the avalanche as the location where the velocity reaches zero. The main disadvantage with using this method is that the friction parameters required for the model are poorly confined, and require experience of the modeller to provide a reasonable estimate of runout. In practice, direct evidence of historical runouts and statistical models are often used to define the extreme runout position in an avalanche path and friction coefficients for dynamics models are chosen to fit the extreme runout. These fitted friction coefficients are then used to estimate avalanche velocities and impact pressures in the runout zone.

The friction values used in avalanche dynamics models are physically based parameters that can vary as a function of numerous factors, including snow properties and path characteristics such as slope angle and surface roughness. True values of friction coefficients for moving snow are difficult to determine using lab experiments (McClung, 1990). The range of friction values given in the literature (e.g. Buser and Frutiger, 1980) has been developed by back calculating values from observed avalanche runout distances, rather than direct field measurements of friction. The large variability between avalanche paths, geographical regions and the type of snow involved complicate the estimation of friction parameters. Transference methods are also used, whereby information from avalanche paths with known runout distances is used to estimate friction parameters for the path of interest by systematically comparing topographical parameters between paths (e.g. Sigurðsson et al., 1998).

Mears (1992, p. 27) noted that statistical analysis has shown that friction parameters cannot be correlated with measurable terrain parameters such as path size or shape. However, Bakkehøi et al. (1981) found that scaling the friction parameter, M/D, in the PCM model (Perla, Cheng and McClung, 1980) with the vertical fall height of the path allowed them to narrow the range of the other friction parameter, μ. They also found that friction parameters could not be directly associated with topographic variables, and scaling with the vertical fall height merely narrowed the range of friction coefficients. These results imply that, by selecting a simple avalanche dynamics model best suited to a dataset of short slopes, terrain parameters may be used to narrow the range of friction coefficient values and provide a first estimate, or average value, of the friction coefficients for the model.

Of the large number of avalanche dynamics models, few can be considered useful for application to short slopes. Many dynamics models include assumptions developed from large avalanche paths (e.g. Salm et al., 1990). Also, many models assume avalanche flow dynamics that may not be well suited to smaller slopes (e.g. avalanche motion as a turbulent fluid) where avalanche speeds are typically slower. Thus, we selected a simple avalanche dynamics model that does not include assumptions for large slopes, and has a relatively simple mathematical formulation. The model proposed by McClung and Mears (1995), the Leading Edge Model (LEM), has several qualities that make it well suited for application to a set of short slopes. First, the model can be simplified to require input of only one basal friction parameter, μ. Thus, there is a unique solution when solving for runout distance in a path. Second, the model calculates avalanche runout for the tip (leading edge) of the avalanche, rather than for the centre-of-mass of the deposit in the runout zone. The third, and perhaps most important reason for selecting the LEM is that it treats avalanche motion as a granular flow (Dent, 1993). It is questionable that the core of dry snow avalanches flows as a turbulent fluid (McClung and Schaerer, 1985; McClung, 1990) and especially questionable for short slopes where speeds are slower. Based on these qualities, the LEM was chosen for analysis with this dataset.

3. METHODS

Field studies included a detailed topographic survey and estimation of the extreme runout position for each path using either vegetative indictors or historical records of extreme avalanches. Similar to earlier studies (e.g. McClung and Mears, 1991; McKittrick and Brown, 1993), the goal of the runout survey was to identify the location of the “100-year” return period event, commonly referred to as the “extreme” runout position. However, the true return period for the extreme runout position likely represents return periods of 30 to 300 years, introducing unaviodable random variation in the data (McClung and Mears, 1991).

Paths were selected in the four mountain ranges based on several criteria, including: vertical fall height; reasonable access by vehicle and foot; well defined path characteristics (e.g. starting and runout zones); well defined extreme runout position; and no run-up on the opposite side of the valley or runout into a water body. Data were collected at 48 short avalanche paths: 16 in the Coast Mountains, 10 in the Columbias, 15 in the Rockies, and 7 paths in the Chic Choc Range or other parts of Quebec. The paths vary from 48°47' to 51°38' North latitude and from 65°55' to 123°10' West longitude. Elevations of the starting zones range from approximately 85 m to 2500 m above mean sea level. Thus, a geographically diverse sample set was obtained in terms of longitude and latitude, as well as elevation. See Jones (2002) for a detailed discussion of field methods.

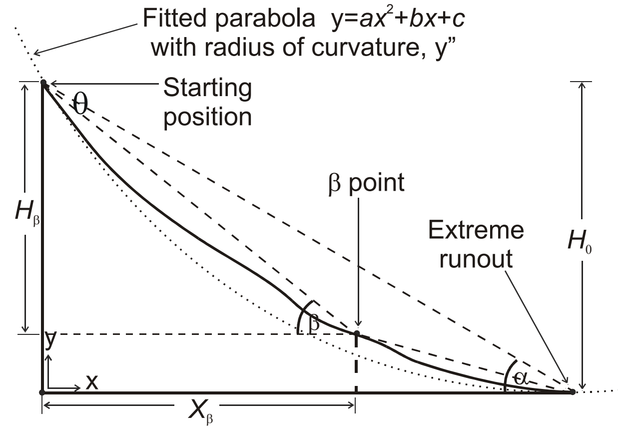

Figure 1. Geometry of example avalanche path showing most terrain variables used in analyses. x-y coordinate system is shown with origin at lower left of figure.

4. DESCRIPTIVE STATISTICS

The data used in this study consist of 14 terrain variables (Table 1). Statistics for the α angle are also shown as these are discussed later in this paper. The variable names are based on previous studies (e.g. Lied and Bakkehøi, 1980; McClung and Lied, 1987), but have been modified where appropriate for this study.

The β angle is defined as the angle (measured from the horizontal) at the β-point to the starting position of the avalanche path (Figure 1). In this study, the β-point is defined as the position at which the slope angle first reaches 24° when proceeding downslope from the starting zone. This differs with the conventional definition of where the slope first reaches 10° (e.g. McClung and Mears, 1991). Jones (2002) provides a statistical argument to support the idea that runout for short slopes is better defined using a β-point at 24°.

The vertical distance to the β-point, H_β, is measured from the top of the starting position to the extreme runout position. The horizontal reach, X_β, is the horizontal distance measured from the top of the starting zone to the extreme runout position.

Table 1. Descriptive statistics for the short slope database.

Variable	*n*	Mean	Standard Deviation	Minimum	Median	Maximum
Beta angle, β (°)	48	32.8	2.8	25.0	32.5	38.5
Vertical fall height to β point, H_β (m)	48	190	125	27	187	634
Horizontal reach to β point, X_β (m)	48	290	176	52	274	894
Alpha angle, α (°)	48	26.5	4.5	18.8	26.6	39.0
Vertical height to low point on parabola, H_0 (m)	48	216	166	28	206	963
Second derivative of the slope function, y'' (m^{-1})	48	0.0023	0.0019	0.00065	0.0018	0.0085
Scale parameter for path profile, H_0y''	48	0.332	0.144	0.071	0.321	0.650
Starting zone inclination, θ (°)	48	38.3	5.0	27.5	38.0	47.5
Starting zone aspect, Aspect (°)	48	139	113	2	97	360
Starting zone elevation, SZ Elev (m)	48	1773	540	85	1890	2490
Runout zone elevation, RZ Elev (m)	48	1480	609	0	1613	2381
Surface roughness, SR (m)	48	0.5	0.4	0.1	0.3	1.5
Wind Index, WI (ordinal data)	48	3.5	1.2	2	4	5
Width of start zone, *W* (m)	48	98	90	17	65	500
Terrain Profile, TP (ordinal data)	48	2.1	0.6	1	2	3

H_0 is defined as the vertical distance measured from the top of the starting position to the lowest point on a polynomial parabolic curve fitted to the terrain profile of a path (Figure 1), the location where the first derivative (slope of the polynomial curve), y′, is zero.

The second derivative of the polynomial curve, y″ (Figure 1), has a value of 2*a* and is the radius of curvature of the path profile (Lied and Bakkehøi, 1980). H_0y″ is the product of H_0 and y″, and serves as a dimensionless scale parameter (Lied and Bakkehøi, 1980).

The starting zone inclination, θ, starting zone aspect and starting zone elevation, SZ Elev, are average values measured in the starting zone. The runout zone elevation, RZ Elev, was an average value measured in the runout zone. The surface roughness is an approximate measure of the height of irregularity in the ground surface, measured in metres. The snow supply available for each avalanche starting zone, *WI*, was categorized in terms of the five-part Wind Index (Schaerer, 1977). The average width of the starting zone, *W*, was measured at the top of the starting zone.

The terrain profile variable, TP, is related to the radius of curvature, y □, but accounts for the very abrupt change in curvature in *hockey-stick* profiles. A value of 1 represents a slope with a nearly linear transition from the track to the runout zone; 2 represents a path with a concave parabolic shape and a relatively smooth transition from the track to the runout zone; and 3 represents a path with a hockey-stick profile. A hockey-stick profile describes paths where there is an abrupt transition to a slope at or near 0□ in the runout zone (Martinelli, 1986).

5. SIMPLIFIED LEADING EDGE MODEL

The Leading Edge Model calculates the stopping position of the tip of an avalanche by solution of one-dimensional momentum and continuity equations (McClung and Mears, 1995). This model assumes that avalanches behave as a dense granular material, and that basal drag is the dominant frictional force. Resistance at the top of the avalanche is included in the model. This model also incorporates a passive snow pressure term that accounts for the slope angle dependence of basal resistance. The model was developed for use in the runout zone where granular flow is expected to be the dominant flow mechanism, and requires an incoming avalanche velocity be specified, usually at the top of the runout zone.

The LEM assumes that avalanche discharge per unit width is constant, which is a reasonable assumption in many runout zones (McClung and Mears, 1995). Many other models assume conservation of mass from the starting position to the runout position (e.g. Perla et al., 1980) which, due to entrainment and deposition of snow in different parts of the path, is likely to introduce larger errors into the model than one that only assumes constant mass in the runout zone.

The force-momentum equation for the Leading Edge Model is expressed as (McClung and Mears, 1995)

$$\frac{\mathrm{d}}{\mathrm{d}t}(vt) = -G_0 t + V - D_0 v^2 t \qquad [1]$$

In Eq. 1 above, *v* is the speed of an avalanche along an incline at time *t*. The resistance term, $G_0 t$, represents the dynamic Coulomb resistance that acts at the base of the avalanche, and G_0 is expressed as

$$G_0 = g\,(\mu \cos\psi - \sin\psi) \qquad [2]$$

where *g* is the gravitational acceleration, μ is the basal friction coefficient, and ψ is the average slope angle in the segment of interest.

The term *V* in Eq. 1 is a momentum loss term that is applied at the transition between slope segments with different slope angles. *V* includes a term to account for the passive snow pressure within a flowing avalanche. In the interest of simplifying the model for practical application (McClung, 2001), the passive pressure term is ignored, resulting in the momentum correction

$$V = v_0 \cos(\psi_0 - \psi) \qquad [3]$$

where v_0 is the velocity of an avalanche entering a segment, ψ_0 is the slope angle on entering a segment and ψ is the average slope angle in the segment of interest.

$D_0 v^2 t$ is the turbulent resistive force applied at the top of the avalanche, and is a function of the density of the snow-dust-air mixture, the average flow density, the drag coefficient and the average flow thickness.

Solving Eq. 1, the velocity at the end of a segment (Point B) can be related to the velocity at the beginning of the segment (Point A) by the simplified expression

$$v_B{}^2 = v_A{}^2 - G_0 x \qquad [4]$$

where *x* is the length of the segment between Points A and B, measured along the slope, and G_0 is the expression shown in Eq. 2. Thus, the velocity of the avalanche can be calculated at each point in the path with knowledge of the incoming velocity at the beginning of the segment, and by applying a momentum correction (Eq. 3) at each slope transition. In the following segment, the velocity at Point *B*, v_B, becomes the initial velocity at Point *A*, v_A, for the next segment, and so on down the profile. All that is required to initiate and apply the model is an estimate of the incoming velocity, v_0, an estimate of the friction coefficient, μ, and a path profile divided into segments of length x_i, each with an approximately constant slope angle, ψ_i.

In the runout zone, McClung and Mears (1995) argue that the turbulent resistive ($D_0 v^2 t$) term in Eq. 1 can be ignored, allowing the equation to be solved analytically. Thus, the runout distance, X_R, in the last segment of the profile is

$$X_R = \frac{V^2}{G_0} \qquad [5]$$

The above simplifications result in a model that can easily be applied to individual paths and solved analytically. Equations [2], [3], [4] and [5] form the fundamental equations for the *Simplified LEM* (McClung, 2001).

The LEM should be applied in the runout zone where granular flow is believed to be the dominant flow mechanism. McClung and Mears (1995) propose that the incoming avalanche velocity be estimated based on velocity data, for which the upper limit may be assumed to be a function of the path slope length, S_0 (McClung, 1990), or the vertical fall height, H_α (McClung and Schaerer, 1993, p. 110). Thus, conditions in the starting zone and track are not important in this model, and modelling typically is initiated in the lower part of the track or top of the runout zone (McClung and Mears, 1995).

6. ANALYSIS

Simplified Leading Edge Models were constructed for each of the avalanche paths in the dataset. Model segments represented the sections of approximately constant slope angle that were measured during the field survey. The last segment in the LEM model was the last surveyed section of the path, of which the end of the segment is the interpreted extreme runout position.

It was decided that one of the fundamental assumptions of the model – that the model is initiated with an estimated velocity at the top of the runout zone – would need to be changed for two reasons. First, avalanche velocities are typically estimated from datasets of velocity measurements from avalanches, which include few short slopes (McClung, 1990; McClung and Schaerer, 1993, p. 110). Some velocity measurements for shorter slopes do exist (e.g. Gubler et al., 1986), but are very limited in number and probably not representative of extreme avalanches. Second, many short slope paths either have a very short track or no observable track, making it difficult to define the location to start the model. The runout would be very sensitive to the assumed starting location.

Table 2. Statistical distribution of the friction parameter, μ, and maximum velocity predicted by the LEM.

	Average μ	Maximum velocity ($m \cdot s^{-1}$)
N	48	48
Mean	0.49	33
Standard Deviation	0.11	8
Minimum	0.29	18
Lower Quartile	0.42	27
Median	0.47	34
Upper Quartile	0.56	39
Maximum	0.80	56

In consideration of the above arguments, it was decided that the model would be initiated at the top of the starting zone (starting position) which is the only known boundary condition for velocity in the path other than the extreme runout position. At both these locations, the velocity of the extreme avalanche is zero, and thus both these locations serve as suitable boundary conditions for the model. It is common practice to initiate other dynamics models at the top of the starting zone (e.g. Mears, 1992, pp. 27, 29, 31), and for practical purposes this assumption was also applied for the simplified LEM, recognizing that entrainment and deposition are neglected.

After setting up the LEM for each path in a spreadsheet, the friction coefficient, μ, was adjusted until the stopping position of the model matched the extreme runout position interpreted from field observations and/or historical records. Thus, a unique value of μ was associated with the extreme runout position for each path. The calculated value of μ can be interpreted to be the mean friction coefficient for the entire path. The constant friction value was applied through the entire path.

Where the extreme runout position and corresponding α angle for a path are known, a first estimate of the average friction coefficient can be obtained using the relationship described by Scheidegger (1973)

$$\mu = \tan(\alpha) \qquad [6]$$

With the range of α in this dataset (18.8° < α < 39.0°) from Table 1, average values for μ range between 0.29 and 0.80, with a mean μ of 0.49 corresponding to the mean α of 26.5°. Table 2 shows the statistical distribution of the average friction coefficients and maximum velocity in the profile calculated using the LEM for each path.

The maximum velocity calculated by the LEM (Table 2) ranges from 18 to 56 $m \cdot s^{-1}$ (mean of 33 $m \cdot s^{-1}$), closely matching the range of typical dry snow avalanche maximum velocity estimates provided by Mears (1992, p. 11) for slopes with a vertical fall height of between 100 and 500 m (20 to 55 $m \cdot s^{-1}$). Paths in the dataset with larger vertical fall heights are associated with avalanche velocities at the upper end of this range. Gubler et al. (1986) used Doppler radar to record the maximum velocity of small (< 500 m^3) avalanches, with speeds ranging between 13 and 28 $m \cdot s^{-1}$. These speed measurements are less than the mean value (33 $m \cdot s^{-1}$) estimates using the LEM probably because the measured avalanche speeds are not representative of extreme avalanches but, rather, represent avalanche speeds in smaller, artificially triggered avalanches. Based on these two sources, the LEM model fitted to the observed extreme runout positions is believed to be providing reasonable estimates of average friction in these paths.

Multiple regression was used to relate various independent predictor variables to the response variable, in this case the average friction coefficient, μ, in the simplified LEM. Fourteen possible predictor variables for μ were chosen for the regression (Table 3).

Spearman rank correlations between the predictor variables and μ are shown in Table 3. Significant variables (p < 0.05) are highlighted. Seven of the 14 variables are significant at this level and these were used to build the regression model. Backward-elimination multiple regression was used with these seven predictor variables to obtain the best fit of the predicted values of μ to the observed values.

Table 3. Spearman rank correlations between the response variable, μ, and the predictor variables used to develop the multiple regression model

Variable	N	R	p[1]
β (°)	**48**	**0.328**	**0.02**
H_β (m)	**48**	**0.681**	**9.8×10^{-8}**
X_β (m)	**48**	**0.662**	**3.0×10^{-7}**
H_0 (m)	**48**	**0.724**	**5.9×10^{-9}**
y'' (m^{-1})	**48**	**-0.340**	**0.02**
H_0y''	**48**	**0.594**	**8.5×10^{-6}**
θ (°)	48	0.112	0.45
Aspect (°)	48	-0.0702	0.64
SZ Elev (m)	48	0.111	0.46
RZ Elev (m)	48	0.0186	0.90
SR (m)	48	0.0315	0.83
WI (ordinal data)	48	-0.104	0.48
W (m)	48	-0.238	0.10
TP (ordinal data)	**48**	**-0.449**	**1.4×10^{-3}**

[1] Rows for which $p \leq$ 0.05 are marked in bold

Table 4. Results of multiple regression for μ.
Adjusted R^2 = 0.76, n = 47, SE = 0.052, $p < 10^{-14}$

	Coefficient β_i	Standard error of β_i	p
Intercept	0.515	0.0295	$< 10^{-19}$
H_0y''	0.578	0.0569	4×10^{-13}
TP	-0.107	0.0125	7×10^{-11}

One significant outlier was identified and removed from the analyses. Variables were then systematically removed from the regression (backward elimination) when they were found to have a minimal effect on the model (i.e. variable F-values less than a specified threshold at each regression step). F-values were computed at each regression step to help facilitate removal of variables from the regression. Using a threshold F-value at the 1 % significance level, all variables but but H_0y'', TP, β and X_β were eliminated from the regression. Additional analyses showed that β and X_β could also be removed from the regression with minimal effect on the results. The remaining two predictor variables in the regression equation were H_0y'' and TP. Removal of either of these two variables from the regression had a strong adverse effect on the model, with adjusted R^2 values dropping from 0.76 using both variables to less than 0.40 when either of these variables was removed. The resulting regression is

$$\mu = 0.515 + 0.578\, H_0 y'' - 0.107\,\mathrm{TP} \qquad [7]$$

This model has an adjusted R^2 of 0.76, a standard error of 0.052, and utilizes 47 of the 48 avalanche paths in the dataset. The regression model has a significance level of 10^{-14}. These two predictor variables are topographic parameters derived from the slope profile and were also used by Jones (2002) in the regression model for estimating α. The similarity of Eq. 9 to the regression equation for α (Jones, 2002) is not unexpected considering the strong relationship between average μ and α. A summary of the regression is shown in Table 4.

7. DISCUSSION

The two independent variables used in the regression model, H_0y'' and TP, are topographic parameters that, unlike α, can be easily measured in the field for every path. While both of these parameters are statistically important to the regression, the physical effect of each variable should be discussed to evaluate their individual contribution to the model.

The scaling parameter, H_0y'' is strongly and positively correlated with μ (Table 3), which means that higher values of H_0y'' are associated with higher friction coefficients in the simplified LEM. Since higher friction coefficients provide more resistance in the dynamics

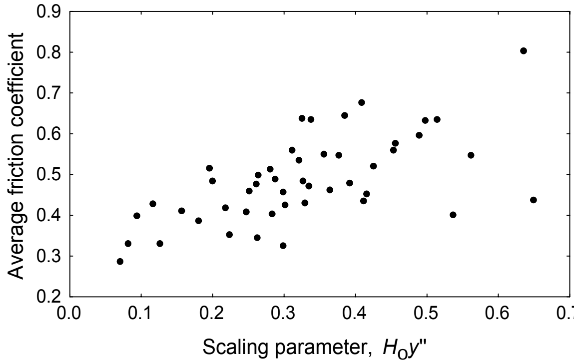

Figure 2. Relationship between the scaling parameter and average friction coefficient, μ, in the LEM analyses

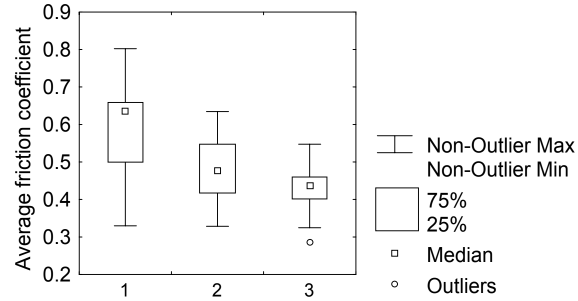

Figure 3. Relationship between the average friction coefficient, μ, in the LEM and the terrain profile variable, TP. Statistics are shown for each range of TP. (1 = Nearly linear/planar; 2 = Concave parabola; 3 = Hockey-stick)

model, they also contribute to shorter runout distances. The relationship between the scaling parameter, H_0y'', and the average friction coefficient in the analyses using the LEM are shown on Figure 2. Highly curved paths (high y''), where greater energy losses are expected, are associated with decreasing slope angles in the runout zone and consequently reduced runout potential (higher μ). The lowest amount of energy loss would be associated with a perfectly linear slope, for which $y'' = 0$. This phenomenon is accounted for in the LEM by applying a momentum correction (Eq. 3) at the transition between segments. On a nearly linear slope $\psi_0 \approx \psi$ in Eq. 6, and thus $\cos(\psi_0 - \psi)$ approaches unity. Thus, only minimal momentum corrections are applied for a nearly linear slope.

The terrain profile variable, TP, is strongly and negatively correlated with μ (R = -0.45, $p = 10^{-3}$) (Table 3). Thus, when terrain parameters are taken from parabolas fitted to path profiles, avalanches in paths with hockey-stick profiles run farther in relation to paths with other profiles.

The relationship of TP with μ is shown in Figure 3, clearly showing μ as a decreasing function of TP. The range of friction coefficients associated with hockey-stick profiles (TP = 3) is quite limited, lying within the range of $0.29 < \mu < 0.55$. Fifty percent of these values lie in the narrow range of $0.40 < \mu < 0.45$. A much larger range of friction coefficients are associated with linear (TP = 1) and concave parabola (TP = 2) profiles. The low values of μ associated with hockey-stick profiles further substantiates the argument that avalanches in these paths may travel greater distances, perhaps due to fluidization upon reaching an abrupt slope transition (Martinelli, 1986; K. Lied, personal communication, 2002), and possibly also a result of material over-riding snow trapped at the slope transition (McClung and Mears, 1995).

8. SUMMARY

Multiple regression was used to estimate the average friction coefficient, μ, in the LEM based on various terrain variables. Average values of the friction coefficient in a path were obtained by fitting the stopping position of an avalanche in the dynamics model to the interpreted extreme runout position from field studies.

The regression providing the best fit for the short slope dataset uses the terrain parameters H_0y'' and TP to predict μ. The predictive model for μ has an adjusted R^2 of 0.76 and a standard error of regression of 0.052, and utilizes 47 of the 48 paths in the dataset.

One of the fundamental assumptions of the LEM, that the model be initiated in the lower part of the track or upper part of the runout zone, was overlooked by initiating avalanche motion at the top of the starting zone. This can be justified when considering that the purpose of this analysis was to develop a useful tool for the practitioner to estimate extreme runout distances, and very few maximum velocity estimates are available for short slopes.

The regression equation developed provides an average value of the basal friction coefficient, μ, to be input into the simplified LEM to simulate avalanche motion on short slopes. This value is only meant to be a first estimate of the friction coefficient, and may need to be subsequently modified based on the knowledge of other terrain and snowpack variables, and interpreted with expert judgment. Also, this value may need to be modified for various parts of the path to reflect changes in terrain and snowpack characteristics.

Hockey-stick profiles were shown to be associated with lower values of the friction coefficient and consequently longer runout distances.

Velocity and runout estimates based on average values of the friction coefficient μ, as developed in this paper, have not been independently verified.

The average friction coefficient can be estimated from α, or, as proposed in this paper, from $H_o y''$ and TP. In contrast to α which can only be determined for paths that do not run-up, and in which the extreme runout is known from historical records or damage to vegetation, the terrain parameters $H_0 y''$ and TP can easily measured in the field for any short avalanche path.

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